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# Understanding Integer Addition and Subtraction Concepts Using Microsoft Word<sup>®</sup> Illustrations

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**Abstract:** *Small colored disks of different colors have long been used to teach integer concepts to middle school children. Concrete drawings of the colored disks may be created using Microsoft Word<sup>®</sup>. This article contains illustrations of integer addition and subtraction problems that may be used in the middle school classroom. Using this mode of instruction, students understand the concepts, are motivated to create mathematical models for problems, and are able to submit their work electronically.*

**Keywords.** *Technology, integers, middle grades, models*

## 1 Introduction

According to the Common Core State Standards Initiative (CCSSI, 2012), Mathematics Standards, Grade 7, “Students extend addition, subtraction, multiplication, and division to all rational numbers . . . students explain and interpret rules for adding, subtracting, multiplying and dividing with negative numbers” (p. 46). The set of integers is a subset of the rational numbers. This article will focus on addition and subtraction of integers.

The National Council of Teachers of Mathematics has set, as one of its objectives, to “understand meanings of operations and how they relate to one another.” In grades 6-8, students should “develop meaning for integers and represent and compare quantities with them,” and “understand the meaning and effects of arithmetic operations with fractions, decimals and integers.” Instructional programs should allow students to “select appropriate methods and tools . . . calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods” (NCTM, 2000, pp. 148, 214).

Multi-colored disks, such as yellow and red disks, are used to teach integer concepts to middle school students in grades 6-7. Instructors should have manipulative disks available for the tactile learner or to review concepts that students have difficulty understanding (Billstein, Libeskind, & Lott, 2013; Long, De Temple, & Millman, 2012).

Microsoft Word<sup>®</sup> was used to draw yellow and red disks. First, circles were drawn using the oval shape under “Insert Illustrations” in Microsoft Word<sup>®</sup>. Disks or circles were shaded yellow or red using the “Shape Fill” option. The yellow and red integer models described in this article are semi-concrete models that most sixth and seventh grade students are able to create. The yellow circles appear white and the red circles appear black on black and white copies.

Students should be introduced to positive and negative numbers in the sixth grade (CCSSI, 2012). The concept of positive and negative numbers may be taught using models such as the number line or a thermometer (Billstein, Libeskind, & Lott, 2013; Long, De Temple, & Millman, 2012, CCSSI, 2012). After students understand the concept of positive and negative numbers, addition and subtraction of integers may be introduced.

## 2 Definitions

In this article yellow disks represent positive numbers and red disks represent negative numbers. Begin with a definition for zero as illustrated in Fig 1. Students should be able to draw the concrete models, give verbal explanations, and write the semi-concrete and abstract equations. After addition has been introduced, students can describe the addition equation and write the abstract equation such as  $(+3) + (-3) = 0$ .

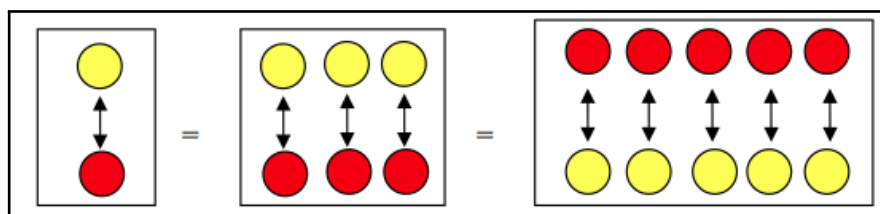


Fig. 1: Concrete Models for zero.

Semi-concrete and abstract equations for zero are “zero = zero = zero” and  $0 = 0 = 0$ , respectively. Zero pairs (1 red to 1 yellow) may be used to create different representations for an integer. Figure 2 shows three different representations for the integer, negative four or  $(-4)$ . The students should be able to explain why each set represents negative four or  $(-4)$ . The students should show other representations for given integers.

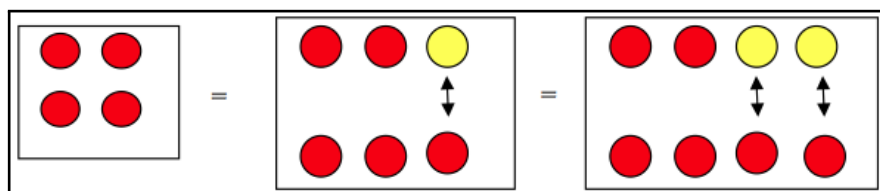


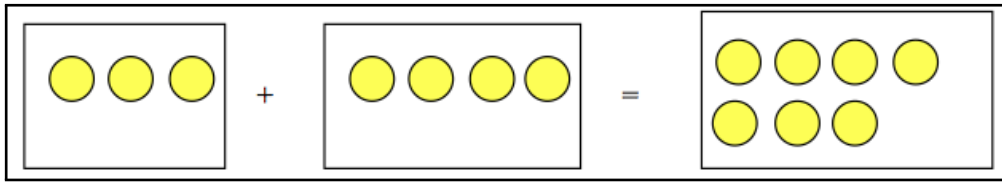
Fig. 2: Concrete models for negative four  $(-4)$

A semi-concrete equation for  $(-4)$  is “negative four = negative four = negative four.” An abstract equation is  $(-4) = (-4) = (-4)$ .

## 3 Addition of Integers

Work as many examples as needed so that students can inductively derive the generalizations involving addition of integers:  $a + b = c$ , where  $a$ ,  $b$ , and  $c$  are integers. In the addition equation,  $a$  and  $b$  are called *addends* and  $c$  is the *sum*. Begin with problems where the addends have the same sign. In Example 1, the addends are both positive, and in Example 2 the addends are both negative. The student should be able to draw the concrete models, give a verbal explanation, and write the semi-concrete and abstract equations for each problem. We provide each in the following examples.

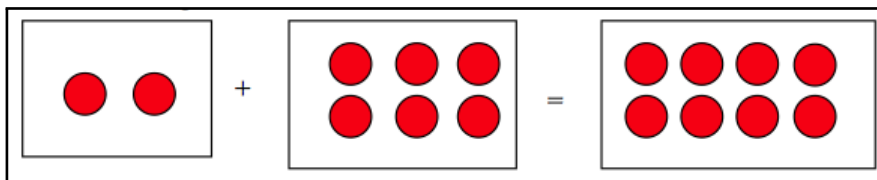
### 3.1 Example 1: Integer Addition - Same Signs, Both Positive



**Fig. 3:** Concrete Model for adding 3 yellow and 4 yellow.

A semi-concrete equation for adding 3 yellow and 4 yellow is “3 yellow + 4 yellow = 7 yellow.” An abstract equation is  $(+3) + (+4) = (+7)$ .

### 3.2 Example 2: Integer Addition - Same Signs, Both Negative



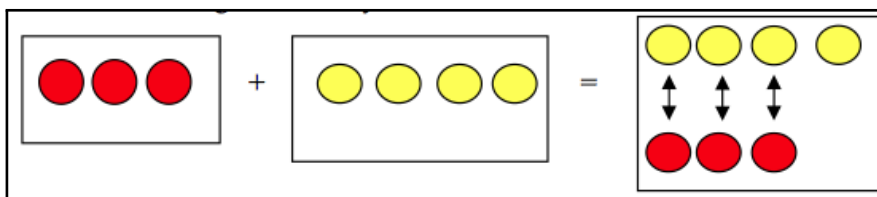
**Fig. 4:** Concrete Model for adding 2 red and 6 red.

A semi-concrete equation for adding 2 red and 6 red is “2 red + 6 red = 8 red.” An abstract equation is  $(-2) + (-6) = (-8)$ .

After students have illustrated several integer addition problems, where the addends have the same sign, the instructor may list several of the abstract equations on the board or overhead, such as:  $(+3) + (+4) = (+7)$ ,  $(-2) + (-6) = (-8)$ ,  $(+5) + (+6) = (+11)$ ,  $(-8) + (-2) = (-10)$ , etc. Next, students are asked to provide a generalization for integer addition with like signs. The students should be able to verbalize a generalization similar to the following: “When adding integers, where the addends have the same sign, add the numbers and keep the sign in the sum.”

After the students have mastered adding integers with like signs, instructors are encouraged to introduce integers where the addends have different signs, such as those provided in Examples 3 and 4. Once again, students are asked to draw the concrete models, give verbal explanations, and write the semi-concrete and abstract equation for the addition problems.

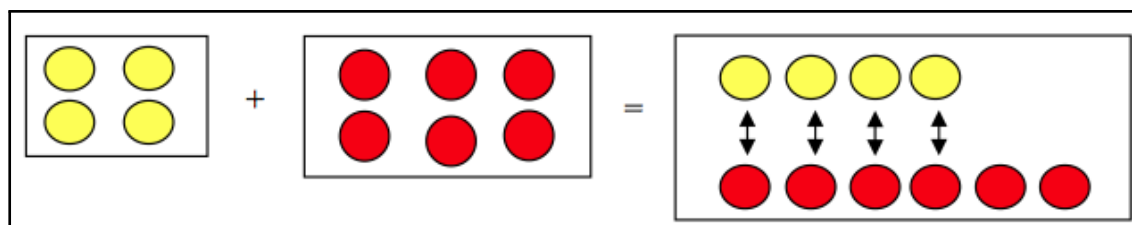
### 3.3 Example 3: Integer Addition - Different Signs, Non-negative Result



**Fig. 5:** Concrete Model for adding 3 red and 4 yellow.

A semi-concrete equation for adding 3 red and 4 yellow is “3 red + 4 yellow = 1 yellow.” An abstract equation is  $(-3) + (+4) = (+1)$ .

### 3.4 Example 4: Integer Addition - Different Signs, Negative Result



**Fig. 6:** Concrete Model for adding 4 yellow and 6 red.

A semi-concrete equation for adding 4 yellow and 6 red is “4 yellow + 6 red = 2 red.” An abstract equation is  $(+4) + (-6) = (-2)$ .

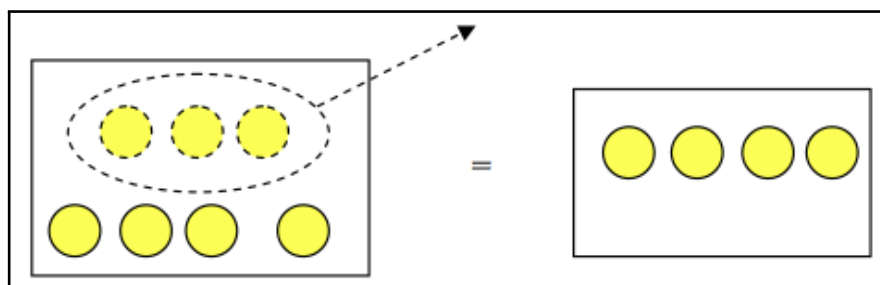
After students have illustrated several integer addition problems, where the addends have different signs, the instructor may list several of the problems on the board or overhead, such as:  $(-3) + (+4) = (+1)$ ,  $(+4) + (-6) = (-2)$ ,  $(+5) + (-8) = (-3)$ ,  $(-8) + (+2) = (-6)$ , etc. Next, teachers encourage students to generalize integer addition for addends with different signs. Acceptable responses include observations such as the following: “When adding integers, where the addends have different signs, subtract the two numbers and keep the sign of the larger addend in the sum.”

Instructors should provide practice of addition problems that use both generalizations. After students have mastered addition of integers, subtraction of integers may be introduced.

## 4 Subtraction of Integers

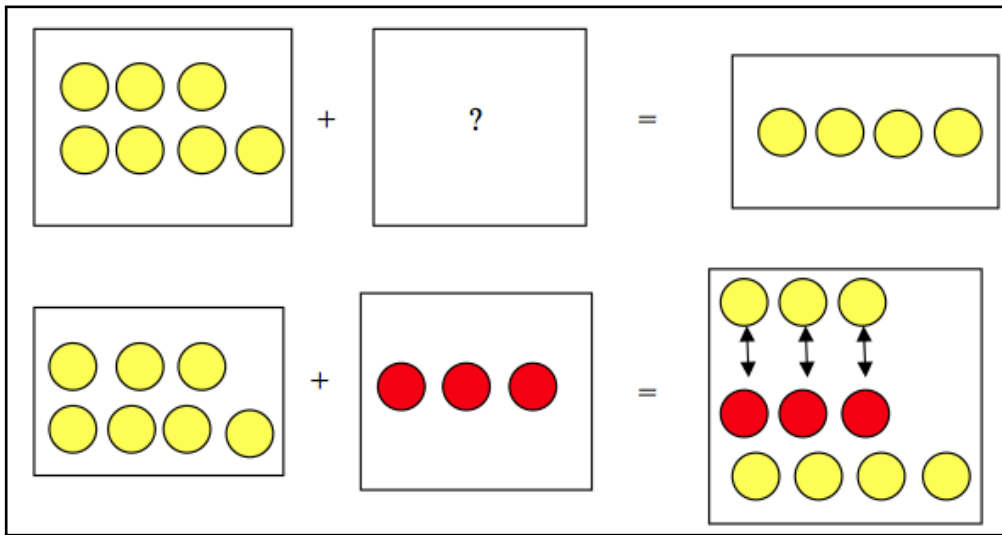
In a subtraction equation,  $a - b = c$ ,  $a$  is called the *minuend*,  $b$  is the *subtrahend*, and  $c$  is the *difference*. Example 5 involves subtracting integers with same signs and Example 6 involves subtracting integers with different signs. Students should be able to draw the concrete models, give verbal explanations, and write semi-concrete and abstract equation for each problem. Students should be guided to: (a) solve the subtraction problem and (b) solve a comparable addition problem.

### 4.1 Example 5: Integer Subtraction - Minuend and Subtrahend have Same Signs



**Fig. 7:** Concrete Model for 7 yellow “take away” 3 yellow.

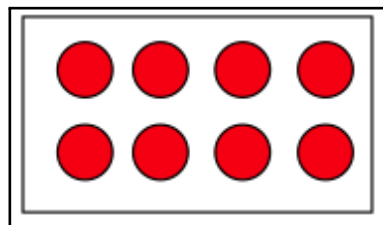
A semi-concrete equation for 7 yellow “take away” 3 yellow is “7 yellow - 3 yellow = 4 yellow.” An abstract equation is  $(+7) - (+3) = (+4)$ . Students operating at an abstract level are asked to solve a comparable addition equation such as 7 yellow + \_\_\_\_\_ = 4 yellow. Concrete thinkers are provided with a comparable task with manipulatives, as illustrated in Fig 8.



**Fig. 8:** *Comparable subtraction task with concrete manipulatives.*

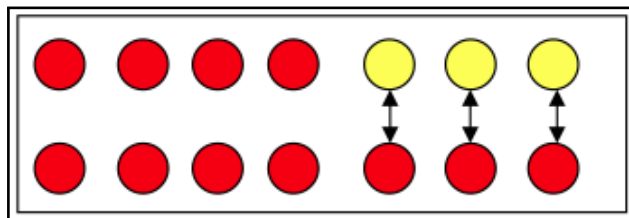
A semi-concrete equation for the comparable task depicted in Fig 8 is “7 yellow + 3 red = 4 yellow.” An abstract equation is  $(+7) + (-3) = (+4)$ . The three zero pairs may be removed from the solution set to show that the four yellow disks remain. After the subtraction and comparable addition problem have been completed, we encourage the instructor to write the two equations from Example 5 on the board or overhead.

#### 4.2 Integer Subtraction - Minuend and Subtrahend have Different Signs



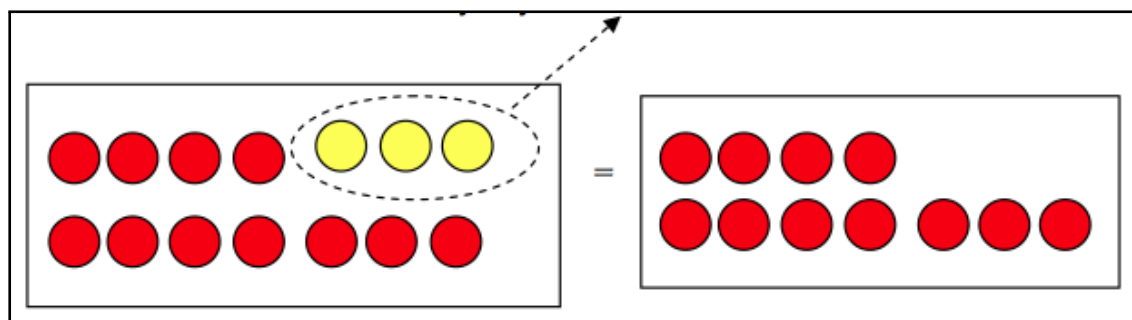
**Fig. 9:** *Initial concrete model for 8 red chips.*

At this point, it is not possible to remove 3 yellow disks from the set of 8 red disks; but this problem can be solved by renaming 8 red. Three yellow and three red disks can be added to the set, since 3 yellow plus 3 red is equal to zero. Fig 10 illustrates a revised concrete model for 8 red.



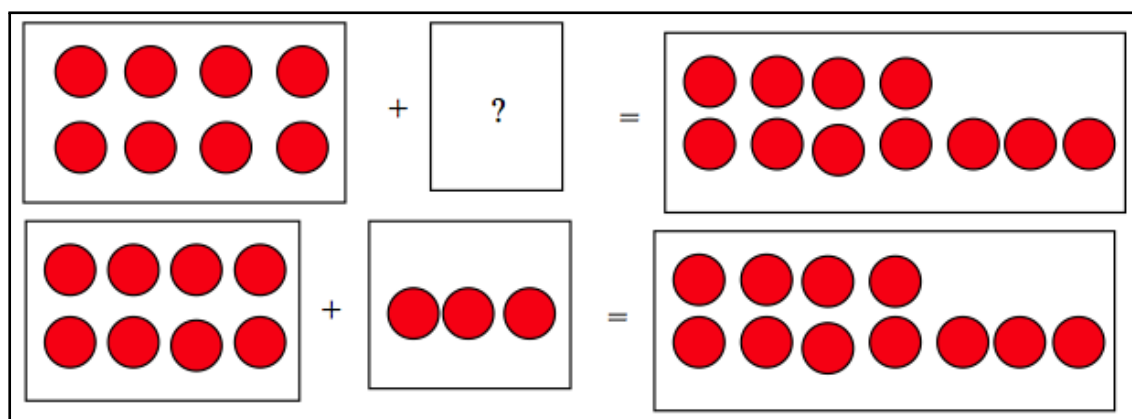
**Fig. 10:** *Revised concrete model for 8 red chips*

With the revised model, students can illustrate 8 red take away 3 yellow as shown in Fig 11.



**Fig. 11:** Concrete model for 8 red “take away” 3 yellow.

A semi-concrete equation for the comparable task depicted in Fig 11 is “8 red - 3 yellow = 11 red.” An abstract equation is  $(-8) - (+3) = (-11)$ . Students operating at an abstract level are asked to solve a comparable addition equation such as 8 red + \_\_\_\_\_ = 11 red. Concrete thinkers are provided with a comparable task with manipulatives, as illustrated in Fig 12.



**Fig. 12:** Comparable subtraction task with concrete manipulatives.

A semi-concrete equation for the comparable task depicted in Fig 12 is “8 red + 3 red = 11 red.” An abstract equation is  $(-8) + (-3) = (-11)$ . After the subtraction and comparable addition problem have been completed, the instructor may write the two equations from Example 6 on the board or overhead. Other integer subtraction problems may be worked with the students. Students should be guided to derive the generalization involved in changing a subtraction equation to an addition equation. In a subtraction equation,  $a - b = c$ ,  $a$  is the minuend,  $b$  is the subtrahend, and  $c$  is the difference. Students should reach a conclusion similar to the following: “A subtraction equation can be changed to an addition equation by changing the subtraction sign to an addition sign and taking the opposite of the subtrahend.”

## 5 Conclusions

Middle school students can use Microsoft Word® to create concrete models for integer addition and subtraction problems such as those illustrated in this article. Students in grades 6 and 7 should be able to illustrate the problems using the red and yellow disks, give verbal explanations of the process, and write semi-concrete and abstract equations for the problems. Students like to use

technology and are motivated to learn the integer concepts. The colored disk manipulatives have long been used to teach addition and subtraction of integers to middle school students (Fierro, 2013 & Van De Walle et. al, 2010). Now, students can use technology, such as the Microsoft Word®, to create and explain the models. Numerous websites provide interactive instruction for integer addition and subtraction. The TicTap Tech (2013) and Utah State University (2010) websites, cited in the references, have interactive manipulatives for learning integer addition and subtraction. The Data Projections (2013) website has information about smartboards, which may be used to draw the manipulatives.

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